## LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034

## M.Sc. DEGREE EXAMINATION - STATISTICS

FIRST SEMESTER - NOVEMBER 2007
ST 1812-STATISTICAL COMPUTING - I

## BB 8

Date : 05/11/2007
Dept. No. $\square$ Max. : 100 Marks
Time : 1:00-4:00
Answer the following questions. Each question carries 33 marks

1. Find the characteristic roots and vectors of the following matrix and also obtain the matrix $\mathbf{U}$ such that $\mathbf{U}^{\mathrm{T}} \mathbf{A U}=\boldsymbol{\Lambda}$ :

$$
\mathbf{A}=\left[\begin{array}{rrr}
6 & 2 & -2 \\
2 & 6 & -2 \\
-2 & -2 & 10
\end{array}\right]
$$

Write the quadratic form associated with the matrix. Find the rank, index and signature.
(OR)
Find the inverse of the following matrix using partitioning method or sweep out process:

$$
\mathbf{A}=\left[\begin{array}{rrrr}
1 & -1 & 2 & -1 \\
-1 & 3 & 4 & 2 \\
2 & 4 & 3 & 1 \\
-1 & 2 & 1 & 1
\end{array}\right]
$$

2. The Transient Point (in volts) of PMOS_NMOS Inverters is believed to depend on the length of PMOS and NMOS devices. Build a model with intercept using the following data:

| Transient <br> Point <br> (volts) | Length <br> of <br> PMOS <br> device | Length <br> of <br> NMOS <br> device |
| :---: | :---: | :---: |
| 0.29 | 8 | 8 |
| 0.20 | 4 | 6 |
| 4.71 | 5 | 5 |
| 9.10 | 4 | 4 |
| 1.37 | 8 | 5 |
| 0.29 | 3 | 3 |
| 9.17 | 8 | 3 |
| 0.38 | 2 | 2 |
| 3.35 | 3 | 3 |
| 0.20 | 2 | 3 |
| 4.97 | 2 | 2 |
| 1.52 | 8 | 6 |

Test for overall significance of the model. Also test for the significance of the individual regressors.

## (OR)

(a) A model with a maximum of four regressors is to be built using a sample of size 30. Carry out 'Backward Elimination Process' to decide the significant regressors given the following information:
$\mathrm{SS}_{\mathrm{T}}=5431.52, \mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{1}\right)=2531.36, \mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{2}\right)=1812.67, \quad \mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{3}\right)=3878.80, \quad \mathrm{SS}_{\mathrm{Res}}\left(\mathrm{X}_{4}\right)=1767.72$, $\mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{1}, \mathrm{X}_{2}\right)=115.80, \mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{1}, \mathrm{X}_{3}\right)=2454.14, \quad \mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{1}, \mathrm{X}_{4}\right)=149.52, \mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{2}, \mathrm{X}_{3}\right)=830.88$, $\operatorname{SS}_{\text {Res }}\left(\mathrm{X}_{2}, \mathrm{X}_{4}\right)=1737.76, \quad \mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{3}, \mathrm{X}_{4}\right)=351.48, \mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}\right)=96.22, \mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{4}\right)=95.94$, $\mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{1}, \mathrm{X}_{3}, \mathrm{X}_{4}\right)=101.66, \mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{2}, \mathrm{X}_{3}, \mathrm{X}_{4}\right)=147.62, \mathrm{SS}_{\text {Res }}\left(\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}, \mathrm{X}_{4}\right)=95.72$
(b) A model with an intercept and two regressors was built using 12 data-points. The observed, predicted and diagonal elements of the Hat matrix are given below. Compute 'Studentized Residuals', plot the normal probability plot and draw your conclusions:

| $\mathbf{Y}_{\mathbf{i}}$ | $\mathbf{Y}_{\mathbf{i}} \boldsymbol{\wedge}$ | $\mathbf{h}_{\mathbf{i i}}$ |
| :---: | :---: | :---: |
| 11.5 | 10.22 | 0.071 |
| 14.88 | 9.57 | 0.085 |
| 18.11 | 20.71 | 0.043 |
| 17.83 | 18.37 | 0.068 |
| 21.5 | 21.90 | 0.196 |
| 21 | 24.72 | 0.114 |
| 19.75 | 21.20 | 0.078 |
| 29 | 35.67 | 0.166 |
| 19 | 16.85 | 0.096 |
| 35.1 | 33.46 | 0.102 |
| 52.32 | 38.42 | 0.392 |
| 19.83 | 28.74 | 0.121 |

3. The distribution of marks secured by students in a particular examination is believed to be a mixture of two normal variates with common variance 25 and equal mixing proportion. Fit the distribution for the following data corresponding to one such distribution.

| Marks | Number of <br> Students |
| :---: | :---: |
| $<10$ | 2 |
| $10-20$ | 15 |
| $20-30$ | 25 |
| $30-40$ | 15 |
| $40-50$ | 30 |
| $50-60$ | 20 |
| $60-70$ | 10 |
| $70-80$ | 5 |
| $>80$ | 2 |

(OR)
(a) Generate Five observations from $N_{2}\left(\left[\begin{array}{l}5 \\ 2\end{array}\right],\left[\begin{array}{cc}10 & -5 \\ -5 & 8\end{array}\right]\right)$
(b) Generate a sample of size 5 from Cauchy distribution with scale parameter 1 and location Parameter 3

